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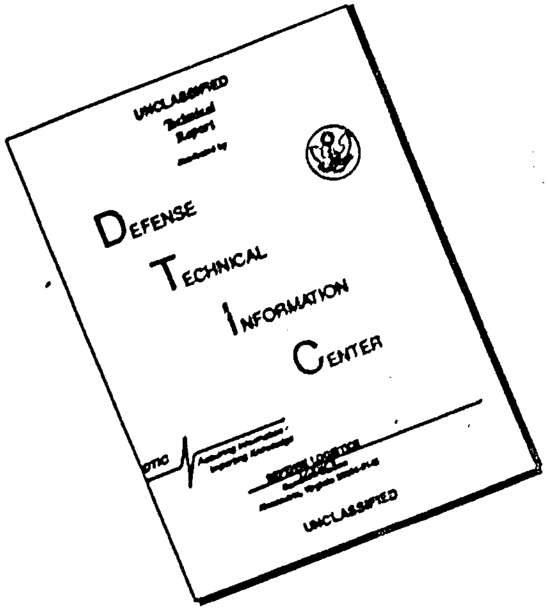

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HYDROMECHANICS

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APPLIED MATHEMATICS

THE ELASTIC BUCKLING STRENGTH OF NEAR-PERFECT DEEP SPHERICAL SHELLS WITH IDEAL BOUNDARIES

by

Martin A. Krenzke

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

July 1963

Report 1713

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ABSTRACT

Two accurately machined hemispherical shells bound by ring-stiffened cylinders designed to provide ideal edge conditions were tested under hydrostatic pressure to determine the elastic buckling strength of near-perfect deep spherical shells. Ratios of experimental collapse pressure to the pressure calculated by using the classical small-deflection theory of 0.73 and 0.90 were obtained. The ratio of 0.90 is considerably higher than the ratios obtained in previous tests recorded in the literature. Since these models had small, unavoidable imperfections, the experimental results lend considerable support to the validity of the small-deflection analysis for the elastic buckling strength of initially perfect spheres.

INTRODUCTION

It is evident that spherical shells will be used extensively in future vehicles designed to operate to great depths. Complete spheres will be used as the entire pressure hull in much the same manner as used in the research vessel TRIESTE. Small spheres may be imbedded in a plastic matrix and used as a float material similar to the present application of glass microballoons. Hemispheres or other spherical segments will terminate the ends of cylindrical or spheroidal hulls.

Spherical shells of conventional pressure hull materials will most likely collapse by inelastic buckling when designed for deep-submergence application. However, shells of nonconventional materials, such as glass and glass ceramics, may collapse by elastic buckling at all depths due to their high compressive strength.[1] In either case, an understanding of the elastic buckling pressure of spherical shells is required to permit rational design procedures.

The classical small-deflection theory for the elastic buckling of spherical shells was first developed by Zoelly in 1915 and has been presented by Timoshenko.[2] Prior to the conduct of recent experiments on machined spherical shells at the Model Basin,[3] very limited data existed and none of these data supported the linear theory. Tests by other investigators,[4] for instance, gave elastic buckling pressures that were only about one-fourth those predicted by classical theory. The specimens used in these earlier tests were formed from flat plate and, consequently, had initial imperfections together with residual stresses and adverse boundary conditions. Until recently,[5–7] however, no attempt was made to theoretically evaluate the effect of imperfection on the collapse strength of spherical shells. Instead, various investigators have attempted to explain the discrepancy between the early experiments and theory by introducing nonlinear shell equations.[8–11]

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[1]References are listed on page 14.

The large discrepancy between the classical buckling pressure and the existing experimental data prompted the Model Basin to test a series of machined shells[3] which more closely fulfilled the assumptions of classical theory. These tests demonstrated the detrimental effects on collapse strength of initial departure from sphericity. The collapse strength of the machined shells was from two to three times greater than the strength of the shells formed from flat plates.

Based on the results of the machined shells, an empirical formula for the elastic buckling strength of near-perfect deep spherical shells was developed.[3] This empirical formula predicts collapse to occur at about 70 percent of the pressure calculated by the classical small-deflection theory. However, it was recognized that this formula is based on tests of shells which, although machined, had small imperfections which presumably had some weakening effects.

To determine more closely the strength of near-perfect spherical shells, two additional shells were machined in which every effort was made to minimize initial imperfections. The test results of these two models are presented in this report.

DESCRIPTION OF MODELS

Two identical structural models, consisting of hemispherical shells each bounded by a ring-stiffened cylinder, were machined from 7075-T6 aluminum bar stock with a nominal yield strength of 80,000 psi. Young's modulus, as determined by optical strain-gage measurements on four specimens, varied between 10.6×10^6 and 11.3×10^6 psi depending on the orientation of the specimens in the bar stock. A Young's modulus of 10.8×10^6 psi is used in all calculations. Poisson's ratio was not measured, but a value of 0.3 is assumed. A sketch of the models, which are designated PS-1 and PS-2, is presented in Figure 1.

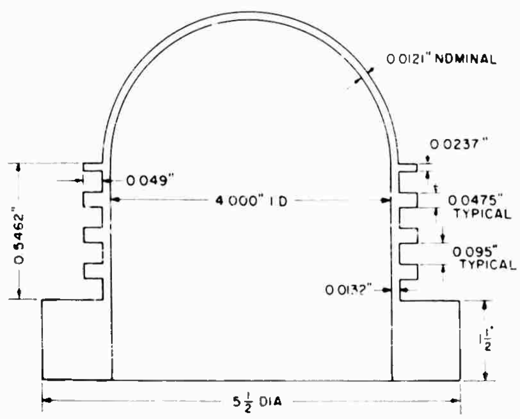


Figure 1 – Models PS-1 and PS-2

The models were designed to study the experimental elastic-buckling strength of spherical shells which closely conform to the idealized assumptions of the classical theory. Therefore, every effort was made to minimize departures from sphericity, variation in thickness, residual stresses, and adverse boundary conditions. In addition, a completely elastic failure was assured by selecting a thickness-to-radius ratio which caused failure to occur at a stress level well below the proportional limit.

Departures from sphericity, variations in thickness, and residual stresses were minimized by careful selection and conduct of the machining processes. First, both the inside and outside contours were rough machined. Then the model was held in place by a pot-type fixture, and the final inside contour was obtained by using a tool specifically designed to accurately generate an inside spherical surface. The final outside spherical contour was obtained by supporting the inside contour by a mating mandrel and by generating the outside surface using a lathe with a ball-turning attachment. Finally, the exterior surface of the ring-stiffened cylinder was machined. Extremely fine feeds together with light cuts were used during the later stages of machining each surface to minimize residual stresses and to produce fine surface finishes. Photographs of Model PS-1 during machining are shown in Figure 2.

The actual wall thickness and initial sphericity were accurately measured for each model. The wall thickness was measured using a support-ball and a dial gage calibrated in 0.00002 in. The initial departures from sphericity were obtained by supporting a dial gage in a jig-bore. These inspection procedures are illustrated by photographs in Figure 2. The measured wall thicknesses and initial departures from sphericity are listed in Table 1. These measurements demonstrate the success achieved through the machining process. The maximum measured variation in wall thickness for each model was approximately 3/4 of 1 percent from the mean thickness. The maximum measured departure from sphericity was less than 1/100 of 1 percent of the mean radius for each model.

The stiffened cylindrical portion of each model was designed to provide conditions of membrane deflection and no rotation at the edge of their hemispherical ends.[12] Thus the hemisphere was designed to behave as a portion of a complete sphere, consistent with the assumptions of classical theory.

TEST PROCEDURE

Foil-resistance strain gages were used to measure strains in both the meridional and the circumferential directions of the hemispherical portion of each model. They were located to indicate the strain distribution near the hemisphere-cylinder juncture as well as in areas far removed from the boundaries. The gage location diagram is shown in Figure 3.

Both models were subjected to external pressure in a small pressure tank. Oil was used as a pressure medium to avoid waterproofing the strain gages. The models were filled with a liquid and vented to the atmosphere to permit measurements of the change in internal

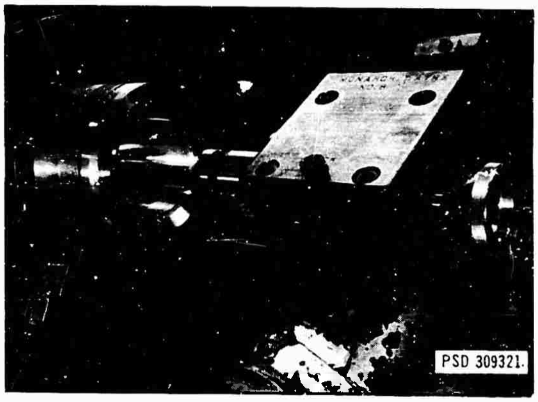


Figure 2a – Generating Inside Contour

Figure 2b – Support Used When Generating Outside Contour

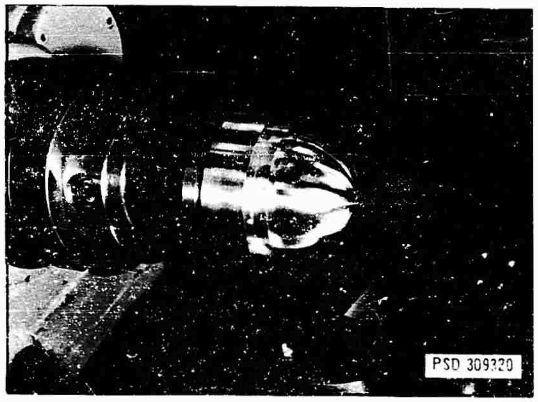


Figure 2c – Generating Outside Contour

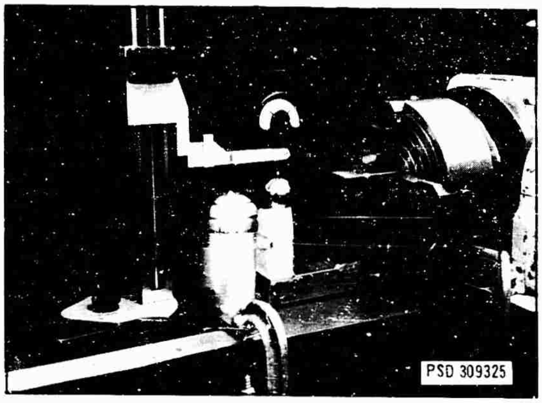


Figure 2d – Setup Used to Measure Hemispherical Wall Thickness

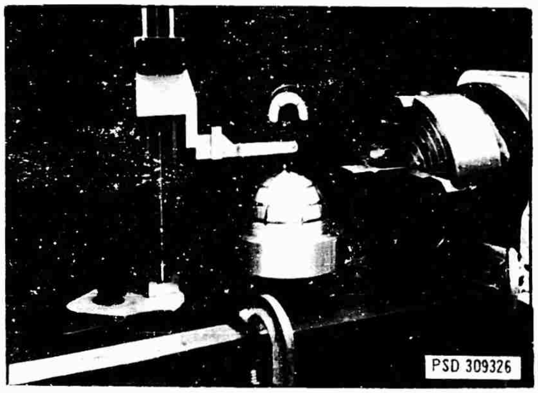


Figure 2e – Measuring Hemispherical Wall Thickness

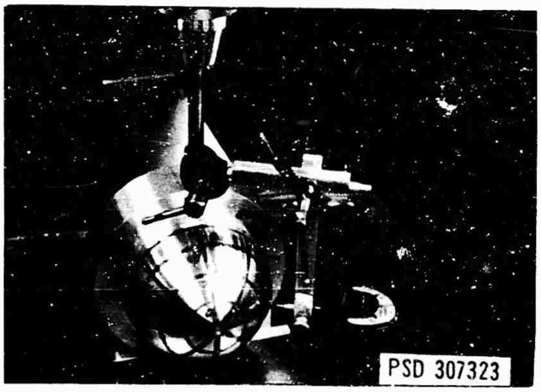
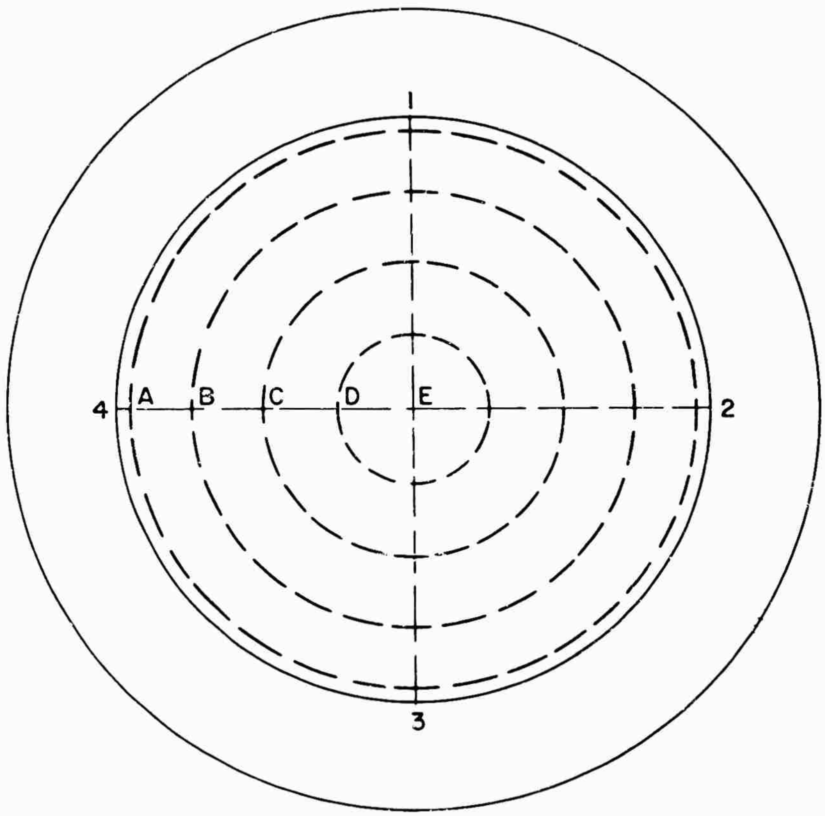


Figure 2f – Measuring Initial Departures from Sphericity

Figure 2 – Model PS-1 during Machining and Inspection

TABLE 1

Measured Wall Thicknesses and Initial Departures from Sphericity



Model	Meridional Orientation	Spherical Wall Thickness, in.					Departure from Nominal Outer Radius, in.				
		Circumferential Orientation					Circumferential Orientation				
		A	B	C	D	E	A	B	C	D	E
PS-1	1	0.01202	0.01204	0.01206	0.01202	0.01200					
	2	0.01208	0.01208	0.01204	0.01204		0	−0.00005	−0.00025	−0.00025	0
	3	0.01214	0.01216	0.01208	0.01208						
	4	0.01210	0.01218	0.01206	0.01204		−0.00005	−0.00015	−0.00025	−0.00025	0
PS-2	1	0.01210	0.01225	0.01220	0.01225	0.01220	0	−0.00020	−0.00020	−0.00015	−0.00010
	2	0.01210	0.01225	0.01220	0.01225						
	3	0.01205	0.01220	0.01215	0.01225		−0.00005	−0.00025	−0.00025	−0.00015	−0.00010
	4	0.01205	0.01220	0.01215	0.01225						

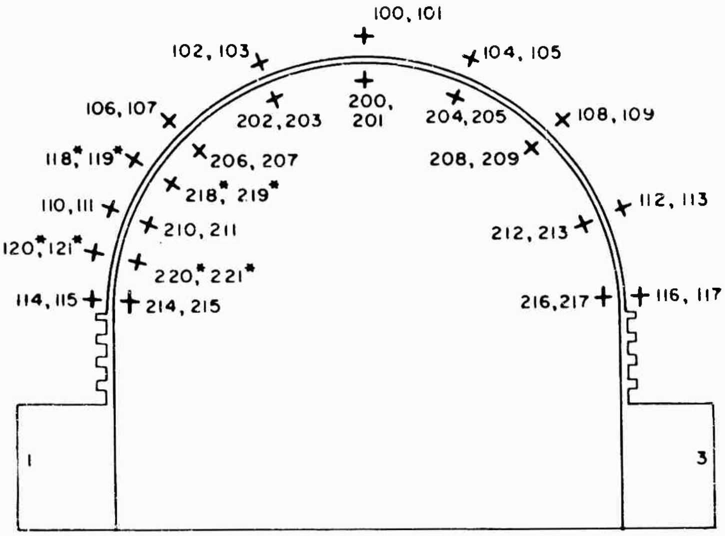


Figure 3 – Strain Gage Locations

Meridional gages are represented by odd numbers,
circumferential gages by even numbers.

*Represents gages placed on Model PS-2 only.

volume during the loading cycles. Photographs of the test setup are shown in Figure 4.

Pressure was applied in increments to both models in three loading cycles. Pressure increments immediately prior to collapse were less than 1 percent of the observed collapse pressure.

TEST RESULTS

Models PS-1 and PS-2 failed at pressures of 347 and 429 psi, respectively. Both failures were confined to a local inward lobe in the spherical portion and occurred in a very sudden, unmistakable manner. Model PS-1 failed in an area somewhat removed from the boundary cylinder and Model PS-2 failed in the spherical portion immediately adjacent to the boundary. Photographs of the collapsed models are presented in Figure 5.

Experimental strain-sensitivity factors, the initial slopes of the pressure-strain plots, are presented in Figure 6 Prior to collapse, no significant nonlinear strain behavior was observed during either of the tests.

Figure 7 presents plots of pressure versus measured change in internal volume recorded during the final pressure loading of each model.

Figure 4a – Instrumented Model Being Placed in Tank

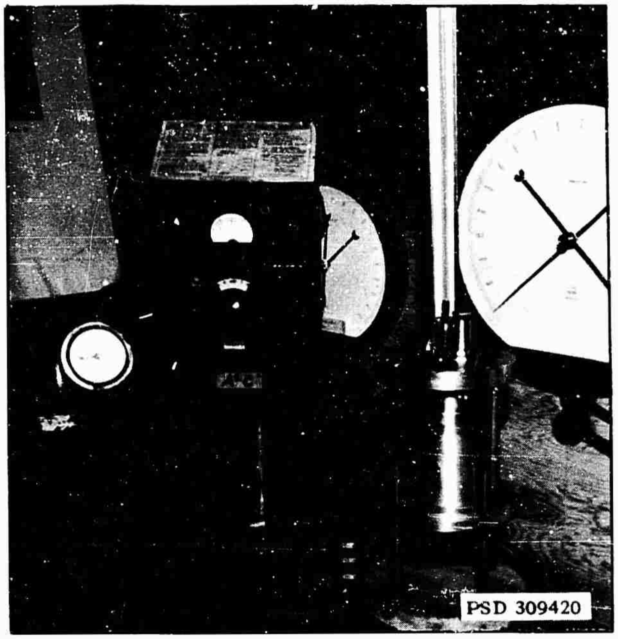


Figure 4b – Final Test Setup

Figure 4 – Test Setup

Figure 5a – Model PS-1

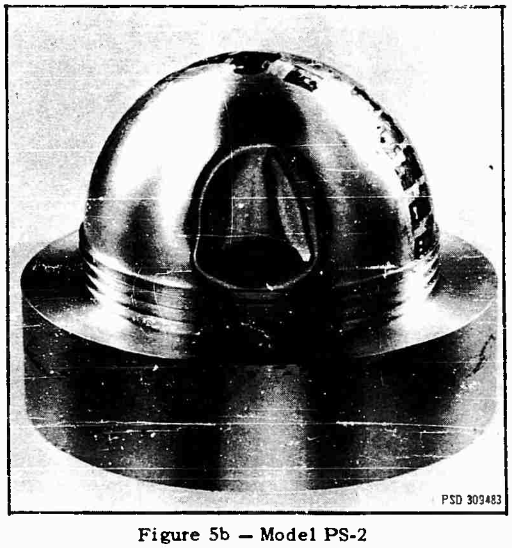


Figure 5b – Model PS-2

Figure 5 – Collapsed Models

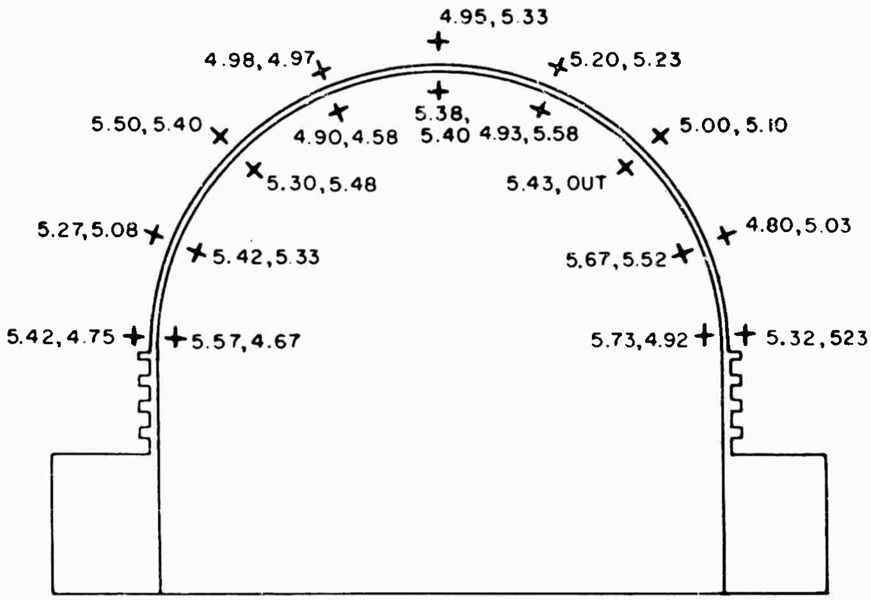


Figure 6a – Model PS-1

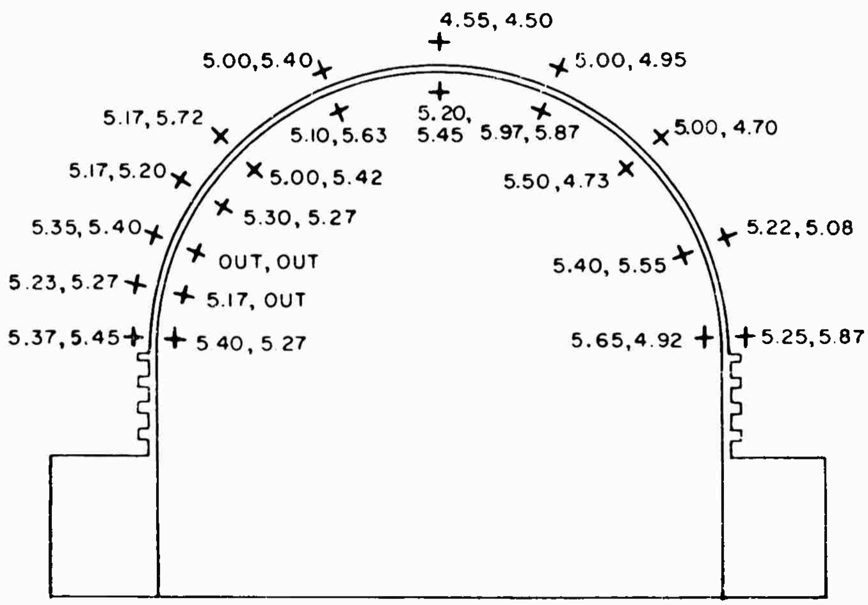


Figure 6b – Model PS-2

Figure 6 – Measured Strain Sensitivities

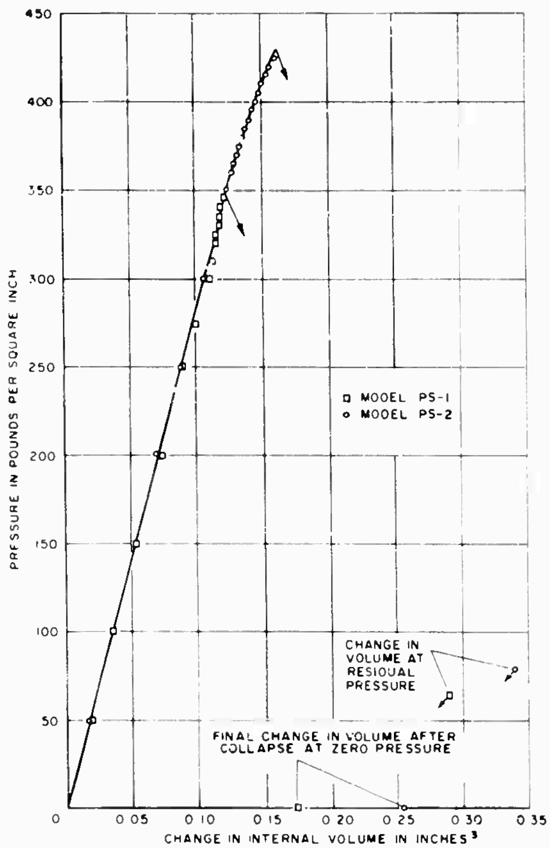


Figure 7 – Pressure versus Measured Change in Internal Volume

DISCUSSION

The ratios of experimental collapse pressure to theoretical elastic-buckling pressure[2, 10] and to the buckling pressure determined from an empirical formula[3] are presented in Table 2. Ratios of experimental collapse pressure to the classical buckling pressure[2] of 0.73 and 0.90 were obtained for Models PS-1 and PS-2, respectively. The ratio of 0.90 achieved in the test of Model PS-2 represents an elastic buckling coefficient of about 1.1, which is by far the highest experimental buckling coefficient obtained to date for deep spherical shells. Both models failed at pressures considerably above those predicted by the nonlinear energy analysis of Tsien.[10] Specifically, Model PS-2 collapsed at a pressure about three and one-half times greater than the lower pressure obtained from Tsien's theory. The empirical buckling equation used in Table 2 is based on the results of recent Model Basin tests of small machined models.[3] Ratios of experimental collapse pressure to the pressure calculated using this empirical equation of 1.06 and 1.30 were obtained for Models PS-1 and PS-2, respectively.

TABLE 2

Comparison of Experimental Collapse Pressures with Theory

Model	Experimental Collapse Pressure p_{exp} psi	$\frac{p_{exp}}{p_1^{*}}$	$\frac{p_{exp}}{p_2^{**}}$	$\frac{p_{exp}}{p_3^{***}}$
PS-1	347	0.73	2.81	1.06
PS-2	429	0.90	3.45	1.30

$^{*}p_1$ is from the classical elastic linear theory of Zoelly.[2]

$^{**}p_2$ is from the elastic nonlinear theory of of Tsien.[10]

$^{***}p_3$ is an empirical buckling pressure for machined spherical shells developed from recent Model Basin tests.[3]

The relatively high collapse pressures observed in these tests are attributed to near-perfect test specimens. By exercising extreme care in the design and manufacture of these models, the idealized assumptions of classical theory were closely approximated. The near-perfect characteristics are demonstrated not only by the initial dimensional measurements tabulated in Table 1 but also by the change in internal volume and strain measurements recorded during the tests. The change in internal volume presented in Figure 7 was essentially a linear function of the applied external pressure prior to the collapse of each model. Similarly, no appreciable nonlinear strain behavior was observed to pressures within 1 percent of the collapse pressure.

Measurements obtained from the strain gages located near the cylinder hemisphere juncture also demonstrate that the cylinders provided ideal boundaries for the hemispheres. Although the repeatability of the strain readings at the various locations was not completely satisfactory, the strain measurements do indicate that the cylinders provided edge conditions of membrane deflection and no rotation for their respective hemispheres. Thus, the hemispheres behaved essentially the same as complete spheres, consistent with the assumptions of classical theory.

Although these models were very accurately fabricated, it must be recognized that it is physically impossible to obtain a completely perfect sphere. As anticipated, therefore, these models had several relatively small shortcomings. For example, the initial measurements show that the wall thicknesses varied approximately 3/4 of 1 percent from the mean thicknesses and that the local radii varied close to 1/100 of 1 percent of the mean radii for each model. These geometric variations are small, and are less than could be expected from even closely controlled machining operations. However, a shell certainly cannot be considered perfect with these known variations in thickness and radii. Furthermore, the nonisotropic material

characteristics, demonstrated by the variations in measured Young's modulus, violate a very basic assumption of classical theory and may be essentially considered an initial imperfection. Finally, the fact that Model PS-2 failed in the hemispherical shell immediately adjacent to the stiffened cylinder suggests that the boundary conditions may have had a slight effect on collapse. This is not surprising in view of the exceptionally high collapse pressure attained by Model PS-2, which indicates the small effects of imperfections on collapse. It should be pointed out, however, that even with perfect membrane boundary conditions, the hemispherical shell is still as likely to fail adjacent to the boundary as in any other area.

By achieving a ratio of experimental collapse pressure to the collapse pressure obtained from the classical theory of Zoelly of 0.9 for Model PS-2, these tests lend considerable support to the validity of the small-deflection analysis for perfect spheres. There was no appreciable difference in the measured initial imperfections present in Models PS-1 and PS-2. Yet, Model PS-2 had a buckling coefficient more than 20 percent greater than Model PS-1. There is no reason to expect, therefore, that the buckling coefficient obtained for Model PS-2 cannot be exceeded if an even more perfect shell is manufactured. Since Model PS-2 collapsed at a pressure only 10 percent less than that predicted by classical theory, it certainly appears reasonable to conclude that the classical solution is valid for spherical shells which satisfy its very rigid assumptions. However, it may be physically impossible to obtain test specimens whose imperfections are so small that they have no influence on collapse strength.

The large increase in strength of these machined shells over the observed strength of fabricated shells is chiefly attributed to the near-perfect sphericity attained by generating the contours. Control of variations in thickness, minimization of residual stresses (which was evidenced by the geometric stability of the models during the final machining processes), and the elimination of any serious boundary effects also contributed to the increase in observed collapse pressures. The contribution of these factors, however, was undoubtedly small since they may be, and have been on occasion, rather well controlled in fabricated spherical shells. Therefore, the single factor which was decidedly different between these machined shells and fabricated shells is the magnitude of the initial departures from sphericity.

Since these results confirm the critical dependence of collapse strength on initial departures from sphericity, they demonstrate also that a design equation must consider the effects of imperfection in order to consistently predict the buckling strength of practical spherical shells. Thus, it appears that most existing theoretical studies have little value in the design of spherical hulls because they do not consider the true reason for collapse at stress levels below the classical buckling stress. The best known example of these analyses is Von Karman and Tsien's work,[8] which suggests that the minimum pressure required to hold an initially near-perfect sphere in the post-buckle position be used to predict the collapse strength of practical spherical shells. Another example is Tsien's[10] "energy criterion of jump" which also assumes an initially near perfect shell but predicts collapse pressure to be a function of the energy, or volume and pressure medium, of the test tank. Both of these analyses predict the same collapse strength for machined and fabricated spheres

whereas the present tests have demonstrated that large differences in strength are obtained experimentally. Specifically, fair agreement has been obtained between the theoretical collapse strength under dead load as predicted by Tsien and the experimental collapse strength of formed spherical shells fabricated within standard commercial tolerances. However, the collapse strength of Model PS-2 was almost three and one-half times greater than the theoretical buckling pressure of Tsien.[10]

A comparison of these test results with previous results of machined models[3] and recent unpublished results of fabricated models obtained by Kiernan[13] at the Model Basin gives a qualitative explanation of the effects of initial departures from sphericity on collapse strength. In general, the tests of machined models have shown that for small, almost unmeasurable, departures from sphericity, the buckling coefficient may vary from a value approaching the classical buckling coefficient to 70 percent or less of the classical coefficient. This was demonstrated by the present tests which yielded experimental buckling coefficients of about 73 and 90 percent of the classical coefficient for shells which had very small initial imperfections of similar magnitude. Thus, it appears unlikely that shells may be machined with sufficient accuracy to justify the use of an elastic buckling coefficient greater than about 70 percent of the classical value for design purposes. In current tests of HY-80 steel hemispheres fabricated by pressing and welding together seven spherical segments, Kiernan has obtained elastic buckling coefficients between about 20 and 40 percent of the classical value. Complete evaluation of the imperfections in these fabricated models has not been completed, but the initial measurements clearly indicated that the departures from sphericity were much less for the models which produced buckling coefficients of about 40 percent of the classical value than for those models which produced coefficients of only 20 percent. Thus, it appears that a relationship exists between collapse pressure and initial imperfections, or an unevenness factor, similar to that illustrated in Figure 8.

As indicated in Figure 8, no single buckling coefficient may be used to consistently predict the collapse strength of practical spherical shells whose initial departure from sphericity and other initial imperfections vary with fabrication procedures. Since most of the initial imperfections, such as variation in thickness, residual stresses, and adverse boundary conditions, can be satisfactorily controlled by proper design and fabrication procedures, it appears profitable to focus attention on the effects of initial departure from sphericity in future theoretical and experimental work. Thompson[6] has recently conducted an "elementary study" of the theoretical behavior of a complete sphere with an initial departure from sphericity. Based on a middle surface imperfection of assumed shape and amplitude, he solved the nonlinear equations for the *maximum* buckling pressure. This type of theoretical approach shows promise of producing the first valid analysis for practical deep spherical shells and appears worthy of further investigation. Tests have recently been conducted at the Model Basin to study the experimental buckling strength of machined shells with known local "flat spots." The results of these tests will be compared with the limited theoretical predictions

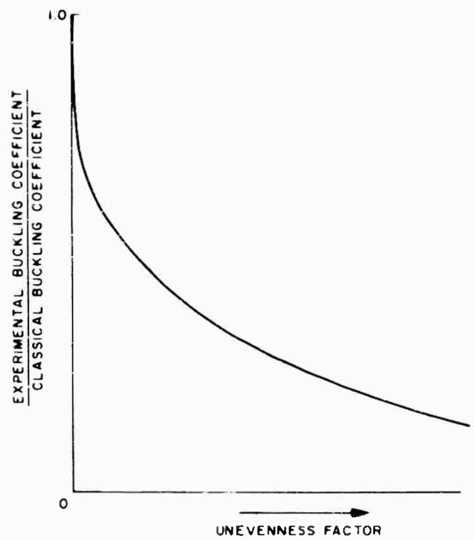


Figure 8 – Effect of Initial Imperfections on the Elastic Buckling Coefficient for Spherical Shells

to help in evaluating the validity of existing theory on the strength of imperfect spherical shells. They will also be compared with these present tests in an effort to provide an experimental relationship between the collapse strength of near-perfect and initially imperfect deep spherical shells.

CONCLUSIONS

1. Ratios of experimental collapse pressure to the pressure obtained from the classical small-deflection theory for the elastic buckling strength of complete spheres of 0.73 and 0.90 were obtained in the tests of Models PS-1 and PS-2, respectively. This difference in observed strength could not be predicted, based on measured initial contours. The ratio of 0.90 is considerably higher than the ratios obtained in previous tests of deep or complete spheres recorded in the literature.

2. By achieving a ratio of experimental collapse pressure to the collapse pressure obtained from classical theory of 0.90, these tests lend considerable support to the validity of the small-deflection analysis for perfect spheres.

3. The ratios of experimental collapse pressure to the pressure obtained from the classical theory which were observed in these tests of machined spherical shells were about two to five times greater than the ratios normally observed in tests of formed spherical shells. This large increase in collapse strength is attributed to the near-perfect sphericity attained in these present models by careful control of the machining processes.

4. A design equation must consider the effects of initial imperfections in order to consistently predict the elastic buckling strength of practical spherical shells.

5. For design purposes, an elastic buckling coefficient of about 70 percent appears to be the upper limit for near-perfect spherical shells.

6. The "lower" buckling pressures obtained from existing large-deflection analyses have no apparent relationship with the observed collapse pressures.

ACKNOWLEDGMENTS

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